Response to Specific Training for Students With Different Levels of Mathematical Difficulties

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ABSTRACT: The purpose of this study was to determine the efficacy of specific, individualized training for students with different levels of mathematical difficulties. Fifty-four students, with either severe or mild math difficulties, were assigned to individualized training or to a control condition. Ten students with severe math difficulties (“dyscalculia”) and 17 with mild math difficulties in the individualized training conditions were trained to improve their accuracy and fluency in math, compared to 9 students with severe math difficulties and 18 with mild math difficulties that were in the general training group (control condition). Students in the individualized training condition (both with dyscalculia and with mild math difficulties) outperformed the control groups after the training and at a later follow-up in almost all math components. Overall, this study supports the feasibility of treating both severe and mild mathematical accuracy and fluency difficulties with specific, customized training.

Mathematics disabilities are identifiable in approximately 5% to 9% of school-age children (e.g., Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996). This proportion is similar to the prevalence of reading disabilities; however, fewer systematic studies have focused on math-related skill deficits (Rasanen & Ahonen, 1995), despite the fact that they are associated with life-long difficulties at school and in the workplace. For example, mathematical competence accounts for variance in employment, income, and occupational productivity even after intelligence and reading have been explained (Rivera-Batiz, 1992).

Presently, in Italy, about five students in a typical class are believed to have mathematical learning difficulties (Lucangeli & Cornoldi, 2007), which means that approximately 20% of students have some level of difficulty with arithmetic. The prevalence of math-related disorders,
based on the Diagnostic and Statistical Manual of Mental Disorders, Fourth Edition (DSM-IV; American Psychiatric Association, 2000) criteria for learning disabilities (LD), however, is lower—only 2.5% of school children have mathematical difficulties (MD) comorbid with other deficits and only 0.5%-1% have arithmetic learning disabilities. In other words, 90% of reported cases of MD have general learning difficulties, not a specific impairment in arithmetic (Lucangeli & Mammarella, 2010).

Students’ attitudes toward math are closely related to their achievement at school; that is, low achievement is associated with negative attitudes toward math.

Why are weaknesses in mathematical skills so common? First, math involves several different components (e.g., calculation, geometry, problem solving) and task requirements vary with respect to these different components. Second, doing math involves specific cognitive processes, including phonological memory (Seitz & Schummann-Hengsteler, 2000), working memory (De Rammelaere, Stuyven, & Vandierendonck, 2001), visuospatial abilities (Dehaene, 1992), and knowledge of strategies. Motivation and affect are further issues—students may be anxious, afraid of failure, or may exhibit learned helplessness (Lucangeli & Scruggs, 2003). Students’ attitudes toward math are closely related to their achievement at school; that is, low achievement is associated with negative attitudes toward math.

Although early prevention strategies can substantially reduce the extent of math difficulties (e.g., Bryant et al., 2011; Fuchs et al., 2005), no prevention program is universally effective. Reviews of the literature (e.g., Gersten et al. 2009; Slavin & Lake, 2008) provide support for a number of practices, including peer-mediated intervention, technology-assisted instruction, strategy instruction, contextualized lessons, and the use of self-monitoring methods. A review of the literature identifies a number of emerging implications for educators, including the following:

1. Effective teaching principles are needed to help students acquire and generalize math concepts and skills (Gersten et al., 2009; Scarlato & Burr, 2002).

2. Teachers should gradually increase the difficulty of mathematical problems to help students progress to abstract levels of math understanding sequentially, from the concrete to the semiconcrete to the abstract (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, Smith, & Jackson, 2003; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel, Mercer, & Miller, 2003).

3. Teachers should promote understanding by employing schema-based instruction to help students represent underlying math structures in order to identify the solution (Xin, Jitendra, & Dealine-Buchman, 2005).

4. Peer-mediated instruction and curriculum-based measurements (CBM) should be used to improve the basic computational skills of secondary students with learning disabilities (Calhoon & Fuchs, 2003).

5. Using video disks to provide contextualized instruction in problem solving appears promising for promoting contextualized problem-solving skills and the ability to generalize based on contextualized problem types (Botte, Heinrichs, Chan, Mehta, & Watson 2003; Botte, Heinrichs, Chan, & Serlin, 2001; Botte, Heinrichs, Mehta, & Watson 2002).

However, an analysis of the literature reveals that many studies have been conducted on populations of typically developing school children (see Slavin & Lake, 2008, for a review). On the other hand, for clinical populations such as children with dyscalculia (DYSC) or MD, there are numerous reports on single cases (e.g., Koeding, Burns & Lukito, 2011) or small groups, but they often have methodological weaknesses such as lack of a random assignment procedure (Slavin & Lake, 2008). Much of the relevant research has been performed on very young children (e.g., Fuchs et al., 2012), and many published studies focus on specific mathematical areas, such as counting (Fuchs et al., 2010) or the mental number line (Kucian et al., 2011). For example, Fuchs
et al. (2010) assessed the effects of teaching strategic counting to students with math difficulties—with and without deliberate practice with counting strategies—on the students’ number combination (NC) skills. In that investigation, 150 students were stratified according to their MD status (MD alone or MD with reading difficulties) and the setting (proximal vs. distal to the intervention developer), and then randomly assigned to control (no tutoring) or one of two variants of NC remediation procedures. Both remediation procedures were embedded in the same validated word problem tutoring protocol (i.e., “Pirate Math”). In Variant 1, the focus on NCs was limited to a single lesson on strategic counting; Variant 2 involved adding 4 to 6 min of practice per session to Variant 1. Tutoring was continued for 16 weeks, with three sessions a week, each lasting 20 to 30 min. Strategic counting lessons with no additional practice (Variant 1) improved participants’ NC fluency compared with controls, but strategic counting with additional practice (Variant 2) produced better NC fluency compared with controls or Variant 1 subjects and also resulted in transfer to procedural calculation.

Kucian et al. (2011) provided computer-assisted training to 16 children with DYSC (8-10 years old) and 16 matched controls for 5 weeks. The purpose of this training was to improve participants’ ability to construct and access the mental number line, starting from the assumption that a spatial representation of numbers is needed to develop mathematical understanding. All the children played the same computer game for 15 min a day for 5 days a week. The results suggested that both groups of children (with and without DYSC) benefited from the training, improving in their spatial representation of numbers and the number of correctly solved mathematical problems. Training also led to a better spatial representation of the mental number line and a modulated neural activation, both of which facilitate the processing of numerical tasks.

These studies underscore the importance of identifying effective and efficient training or teaching practices to help children with DYSC or MD, but there is still little general consensus, or any specific and clear guidelines, on how to proceed. Most reports on such practices in the international literature contain recommendations on what to do once a diagnosis has been established or suggest programs for specific calculation processes (for a review, see Gersten et al., 2009) rather than providing guidelines inspired by practical models or analyses on the cognitive processes behind calculation. There are also no studies comparing different levels of severity of MD; instead, much of the research conducted has focused on different types of MD, particularly on its association with reading disabilities. For example, Powell, Fuchs, Fuchs, Cirino, and Fletcher (2009) assessed the efficacy of fact retrieval tutoring in children with MD alone, compared with children who had both mathematical and reading difficulties.

Presently, there is no conclusive evidence of superiority of one of several methods of training, or of the specific characteristics that a valid, effective training method should possess. The purpose of the present study, therefore, was to investigate the nature and focus of effective math training for assisting elementary school students with MD or DYSC.

In light of the present literature, we intended to compare two types of mathematical training—specific individualized training versus general scholastic training—for children with different levels of MD. According to the literature, a distinction can be drawn between DYSC and MD in terms of the severity and pervasiveness of a child’s difficulties in various mathematical skills. Children with DYSC do not perform as well as children with MD in tasks involving, for example, number comparisons, fact retrieval, or mental and written calculations, and they usually do not respond as well to intervention. Children with MD may reach age-appropriate levels of achievement after a period of specific mathematical training, although this is thought to be less likely in cases of DYSC (Landerl, Bevan, & Butterworth, 2004; Lucangeli & Mammarella, 2010; Mazzocco, 2005).

The design of our study enabled us to assess the efficacy of specific training for school children with different levels of MD. Our research questions were:

1. What is the efficacy of a specific training program compared to general training in support classes for children with DYSC?
2. What is the efficacy of a specific training program compared to general training in support classes for children with MD?
3. What clinical changes are generated by the two types of training in both children with DYSC and those with MD?
4. What is the 4-month maintenance of the specific training program?

**METHOD**

**PARTICIPANTS**

The sample consisted of 54 students attending the second to fifth years of elementary school. Nineteen students had a diagnosis of DYSC and 35 had MD. More specific information is provided under the “Math Disability Classification Criteria” subheading.

The children were stratified by gender, school year, and age, and randomly assigned to two treatment conditions: 27 students in the individualized training condition (hereafter referred to as the experimental condition) and 27 in the control training condition (hereafter referred to as the control condition). Random assignment resulted in the following 4 groups:

1. DYSC experimental group: 10 students with dyscalculia who received the individualized training.
2. MD experimental group: 17 students with math-related difficulties who received the individualized training.
3. DYSC control group: 9 students with dyscalculia who attended the control training program.
4. MD control group: 18 students with math-related difficulties who attended the control training program.

For all students involved in this investigation, we received appropriate approvals from parents and school. All students were Caucasian and had no physical, sensory, or neurological impairments; their intellectual abilities were within the average range according to the Wechsler Intelligence Scale for Children (WISC III; Wechsler, 1991), and they spoke Italian fluently. According to their teachers, each of our participants had grown up in an adequate sociocultural environment. Table 1 summarizes the children's distribution across the 4 school years, by gender and age, in the two treatment conditions. All the students' IQ scores were assessed using the WISC III, and no statistically relevant differences emerged between the two groups (experimental condition, $M = 98.8$; control condition, $M = 96.4$).

**DEPENDENT MEASURES**

Two of the most widely used Italian test batteries, the AC-MT (Cornoldi, Lucangeli, & Bellina, 2002) and the ABCA (Lucangeli, Tressoldi, & Fiore, 1998), were used to assess the students’ mathematical skills. The AC-MT is a battery for assessing calculation ability; it is a paper and pencil tool used for screening in schools and clinical settings. Test-retest reliability of the AC-MT is $r = .65$ (mean for all subtests). The ABCA is another battery for the assessment of mathematical ability and provides a specific profile identifying each child's calculation components resulting above the cutoff criteria with respect to the normative sample at baseline. Internal consistency of the ABCA is $\alpha = 0.78$ for accuracy and $\alpha = .87$ for speed; test-retest reliability is $r = .66$; construct validity ranges from $r = .52$ to $r = 0.66$ for accuracy, and from $r = 0.51$ to $r = 0.76$ for speed. The most significant subtest measures are described in the following paragraphs.

**Mental Calculation.** Students are asked to perform mental calculations (six operations: three addition and three subtraction). For each operation, the time taken is measured from the moment the examiner finishes saying the numbers in the operation aloud to the moment when the child answers. The time limit for each calculation is 30s. The operator asks the students what strategies they used and records their response, for example rounding numbers or decomposition strategies (e.g., $5 + 8 = 5 + 5 + 3 = 13$). Two parameters are considered for this task, time and number of errors.

**Written Calculation.** This subtest examines the child's application of the procedures needed to complete written computational operations (addition, subtraction, multiplication, and divi-
sion), and the degree of automaticity involved. Responses are scored for correct answers.

Arithmetical Facts. This task is used to investigate how students have stored combinations of numbers and whether they are able to access them automatically, without purposive calculation procedures. The items include addition, subtraction, and multiplication, presented verbally and allowing 5s to answer for each of the 12 items. Examples of arithmetical facts are simple operations such as $6 - 3, 8 + 2$, and $10 - 5$. Responses are scored for number of errors.

Numerical knowledge includes the following subtasks (scored as the sum of the correct answers):

- Number comparison. Six pairs of numbers are presented and students are asked to circle the larger number of each pair, for example: “Which number is larger?” (e.g., 12 vs. 36 for second grade or 856 vs. 428 for fourth grade). This task requires both an understanding of the semantics of numbers and the ability to read numbers (lexical level).

- Transcribing digits. This task assesses students’ ability to elaborate the syntactic structure of numbers that governs the relationship between the digits the numbers contain. Students are shown six series of verbally described numbers and are asked to transform them into a final number. For example, for the fourth-grade level, the examiner states, “We have 3 tens, 8 units, and 2 hundreds,” and asks the student to transform that into the associated number (43).

- Number ordering (from greatest to least, and least to greatest). This task is used to assess the semantic representation of numbers by means of quantity comparisons. To answer correctly, the child must be able to recognize single quantities, compare them, and place them in the correct order by magnitude. Five series of four numbers were presented (e.g., $15, 58, 36$, 7 for second grade or 36, 15, 576, 154 for fourth grade), and the student is asked to arrange each series in the correct order.

### Math Disability Classification Criteria

Students’ diagnoses were established by qualified child psychiatrists or clinical psychologists following the guidelines in the Consensus Conference on Learning Disabilities (2010) and the DSM-IV-Text Revision (TR):

A. Mathematical ability, as measured by individually-administered standardized tests, is substantially below that expected given the person’s chronological age, measured intelligence, and age-appropriate education.

B. The above-mentioned impairment in Criterion A significantly interferes with academic achievement or activities of daily living that require mathematical ability.

C. If a sensory deficit is present, the difficulties in mathematical ability in excess of those...
usually associated with it. (APA, 2000, Section 315.1, pp. 53-54)

Students were classified with DYSC if they scored at least 1.5 standard deviations below average for accuracy or speed in at least four of the six areas measured using the AC-MT 6-11 test (the AC-MT test designed for 6- to 11-year-olds; Cornoldi et al., 2002), that is, mental calculation, written calculation, counting, number dictation, arithmetical facts, and numerical knowledge (which in turn includes number comparison, transcribing digits, and number ordering). Students scoring at least 1.5 standard deviations below average for accuracy or speed in no more than three of the above six areas were classified with MD. Overall, DYSC students’ z scores were $M = 1.84$ ($SD = 1.09$) for accuracy and $M = 4.23$ ($SD = 2.34$) for time; MD students’ z scores were $M = 1.22$ ($SD = 1.15$) for accuracy and $M = 1.81$ ($SD = 1.92$) for time.

**Procedure**

All students were assessed at a research and service center dedicated to the assessment and treatment of LD in northern Italy. Each child’s mathematical learning was assessed in a quiet room by a psychologist specializing in LD.

The treatment was provided individually at the center for the experimental condition and at school for the control condition. The specific individualized (experimental) training was provided by psychologists specializing in the treatment of learning disabilities; the control training was provided by educators. Both psychologists and educators were observed and supervised by one of the authors every 4 weeks. In the experimental condition, tasks were differentiated and adapted to each student’s individual difficulties, based on the assessment of their mathematical learning profiles and taking the curriculum followed by their teachers into account. In the control condition, activities represented an extension of the math education program students received in school. These involved the same areas of experimental training; however, activities were differentiated by grade level and not tailored to the mathematical learning profile of each child.

The study design involved the following phases:

1. **Learning level assessment.** This phase involved defining each child’s learning profile, emphasizing the main areas of mathematical learning difficulties and their general cognitive abilities and emotional and motivational aspects related to learning.

2. **Baseline** (analysis of individual profiles and treatment planning). All the individual learning profiles were assessed to select the areas of greatest deficiency on which to focus training.

3. **Training.** The structure of the training was the same for both conditions (experimental and control). Sessions were planned for 32 weeks (with at least one follow-up assessment after 4 months). The first cycle of training was provided twice weekly for at least 4 months (September to December); the second cycle once a week (from January to May). The sessions lasted 75 min each (with a short break about every 15 min).

4. **Posttraining assessment** (efficacy analysis). Posttreatment assessments were collected over a 1-week period for each participant.

To test the validity of the training, criteria were needed to evaluate any improvement in the various areas of calculation. The following criteria were identified, based upon the guidelines of the Consensus Conference on Learning Disabilities (2010): the primary goal was to increase the accuracy index by at least 1 standard deviation and/or to reduce (by 1 standard deviation) the time taken to respond in at least half of the deficient components (mental calculation, written calculation, arithmetical facts, and numerical knowledge).

**Structure of the Individualized Training**

An initial assessment provided a clear profile of each child’s abilities and deficits in specific learning areas. Then, the psychologist organized activities intended to promote competence in the subsequent level, to progress step-by-step, respecting the child’s specific competences. This method derives from Vygotsky’s zone of proximal development concept (1931), defined as the distance between the actual developmental level as determined by independent problem solving, and
the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. In our work, we started with the areas in which each of the children had difficulties and developed specific activities designed to strengthen the abilities they needed to improve their performance in that particular area. We also began with exercises that contained a substantial amount of scaffolding, which was gradually removed.

At the end of the experimental, individualized training, its effectiveness was assessed by repeating the initial assessment to identify (again by comparison with normative data) any specific skills acquired in the initial area(s) of impairment.

The individualized training had two fundamental goals: one was to enable the students to achieve a sufficient level of accuracy; the other was to improve their speed of response. The training was assessed in relation to the gains in the following fundamental calculation skills:

1. Concepts of number (numerical knowledge).
2. Automaticity in retrieving and using arithmetical facts.
3. Mental calculation.
4. Written calculation.

**1. Concepts of number (numerical knowledge),** including semantics (comprehension of quantity), lexicon (reading and writing numbers), and syntax (positional value of digits comprising a number). In this first individualized phase, Arabic numerals were always combined with the quantities to which they referred (also using analogical representations of quantity). Work on the *numerical lexicon* (enabling students to acquire skill in naming the numbers) was associated with the lexicon relating to the function of the signs of the operations and greater than and less than signs (>, <). Learning to name the numbers up to a thousand or more involves discovering the rules for attributing verbal labels. Therefore, the focus was on integrating the various aspects (name, number, and quantity) of the numbers before turning to calculation (Lucangeli, Tressoldi, & De C Candia, 2005). Acquisition of correct reading, writing, and naming Arabic numerals verbally was always associated with progressive and regressive counting (i.e., by ones, threes, fives, and tens). The students were given assistance reading and writing numbers and trying to overcome any difficulties associated with order of magnitude and complex elements such as zero or phonologically similar figures (e.g., six and seven) that can sometimes interfere with reading or writing the numbers correctly. Work on the semantic aspects of mathematics included exercises that involved switching from the analogical representation of a number to the corresponding Arabic numeral, and vice versa, transforming the Arabic numeral into an analogical representation of the corresponding quantity. The quantification process was stimulated by tasks involving estimating quantities, initially using analogical-intuitive material or visual presentations of number values (representing the quantity corresponding to the number). Attention then focused on developing the child’s comprehension of numerical equivalents by using appropriate quantifiers (e.g., “as many . . . as . . .”). When reading and writing numbers 10 and higher, students were taught to understand the relevant syntax. The types of exercises relating to this aspect focused on helping the child to understand the function of the digit’s position, which changes the name and value of the number because, in our numerical system, the verbal label for each digit in a number reflects the order of magnitude defined by the digit’s position. The students were also taught to practice using syntactic markers (e.g., the “comma” indicating the “thousands” element) with the aid of considerations that rely on metacognitive rather than procedural elements; in other words, we tried to stimulate a real recognition of the importance of the position of the digit in the number in representing its value.

**2. Automaticity in retrieving and using arithmetical facts,** that is, basic operations that need not be calculated because they are already known (e.g., “3 + 2” or “3 x 4”) and can be recalled immediately. The goal of this section was to reduce the burden, in cognitive terms, of completing calculations. In this phase of the training, arithmetical facts are seen as being fundamental to the acquisition of calculation skills, considered as reference nodes for fluidly and correctly solving more complex calculations. Training involves children first understanding the significance of a fact and then developing automaticity of recall, be-
cause we did not want children to learn by heart (like a nursery rhyme) without understanding, but we wanted them to understand the facts and then memorize them (Poli, Molin, Lucangeli, & Cornoldi, 2006). The first steps of acquiring the facts were facilitated by associating them logically, which avoided prompting erroneous answers and exploited the organization of semantic memory. The subsequent repetitions enable the facts to be fixed in the child’s semantic memory and consolidated so that their recall becomes highly automatic. The arithmetical facts are presented to the child in different settings with different types of reasoning or exercises during subsequent sessions.

3. Mental calculation. To introduce the child to mental calculation, the first step involved subitizing (totaling small numbers of objects without directly counting) using the analogical approach and Arabic numerals. Sets of five items with a constant spatial arrangement were provided and the students were taught to perform addition and subtraction using not only their ability to count (n + 1 or n-1), but also small groupings (in fives, twos, and threes) with the aid of the perceptual reference (Lucangeli, De Candia, & Poli, 2004; Lucangeli, Poli, & Molin, 2003). Rapid calculation strategies were practiced to increase the students’ calculation speed and to develop automaticity, such as adding up starting from the largest number. Strategies were systematically and gradually taught for rounding numbers off to the nearest 10, or for breaking down and combining numbers.

4. Written calculation. In this stage of the training, exercises were presented on the rules governing columns of numbers, calculation using the four arithmetical operations, and regrouping. Students were taught using simple numbers first, and followed by greater, more complex numbers (Lucangeli et al., 2003; Lucangeli et al., 2004).

Training was provided on each of the various processes in need of remediation, but as part of a coordinated whole, which enabled action to be taken selectively, focusing on specific calculation difficulties. During training, different methods were provided for accessing and coding a number (in its phonological, visual, and analogical aspects). Each child followed a plan that focused on developing their metacognitive components, favoring metacognitive training characterized by the retrieval of experiences related to a task, the exploitation of the child’s individual cognitive strengths, and the acquisition of number sense. For example, at the end of the session, the children had to summarize what they had learned and explain how it could be generally applicable to other contexts.

The instructional sessions were organized as follows:

- Presenting the task and explaining the goal, considering the various methods for accessing and coding the number (using phonological, visual, and analogical pathways, giving students the chance to use the approach they find most congenial to their comprehension of the task), with a view to enabling the students’ understanding of the meaning of the activities.
- Working on the material, presenting the various strategies that the students can adopt or modify to suit their needs (constantly referring to their independent management of their learning processes).
- Discussing and comparing the strategies, with the operator’s guidance (to reinforce the metacognitive component).
- Summarizing the work done (in essential terms), first by the child, then by the operator.
- Conducting a self-assessment by the students, considering metacognitive and motivational components.
- Practicing each strategy the students have learned in a coordinated, continuous, and contextualized manner (with consolidation exercises to complete at home).

During the entire procedure, it was considered particularly important to integrate procedural aspects of the learning tasks with reasoning and metacognitive processes. This was done to orient the students to the meaningfulness of the activities as a useful support for developing their arithmetical competence.

CONTROL CONDITION TRAINING

In the control condition, students were provided with an equivalent amount of time working on
topics that were the same as those in the experimental condition (i.e., concepts of number, automaticity in retrieving and using arithmetical facts, and mental and written calculation tasks). In this condition, however, the activities were related more to the grade-appropriate curriculum than to the specific needs of individual students. That is, in the control condition, students completed assignments differentiated by grade level. Students were provided with additional time to complete exercises of the same type as those given in school and to complete their mathematics homework with educators. Educators could explain some simple strategies, such as the use of grids to facilitate alignment procedures in written calculation. Students in this condition completed general activities related to the principal topics of calculation. To ensure that the educators kept to the training topics and effectively served in a supporting role, these activities were monitored and supervised by one of the authors.

Fidelity of Implementation

During the experimental condition training, the trainer maintained a daily journal of activities undertaken in each session. In addition, a written record was maintained of the observation and supervision sessions of the control condition, considering the topics of each implementation session. In each case, observed activities corresponded approximately $r = .90$ to intended components of the lessons.

Data Analysis

Dependent variables for the following analyses included mental calculation (number of errors and time), written calculation (number of correct answers), arithmetical facts (number of errors), and number knowledge (total score from three subtasks). We analyzed the data using a group (experimental vs. control group) by time (pre- vs. posttraining) analysis of variance (ANOVA), separately for DYSC and MD groups. In addition, we analyzed the results with a clinical approach. Predefining a positive change of at least one standard deviation to represent clinical improvement, we considered the percentage of participants in each group who met this criterion. Finally, a small group of students who received the training underwent a follow-up assessment, and their performances were compared at the three different time points.

Results

Efficacy of the Treatment in Children With Dyscalculia

Results were analyzed separately for the groups of students with DYSC and MD. In Table 2, descriptive statistics are provided for the DYSC group. The two (time) by two (condition) ANOVA yielded significant results on mental calculation errors for time (pre-post), $F(1,17) = 16.4, \eta_p^2 = .49$; condition (experimental-control), $F(1,17) = 7.72, \eta_p = .31$; and interaction, $F(1,17) = 6.64, \eta_p = .28$. For mental calculation time, we observed nonsignificant main effects, but a significant effect for interaction, $F(1,17) = 22.9, p < .001, \eta_p^2 = .57$. For written calculation, we observed a significant main effect for time only, $F(1,17) = 20.06, p < .001, \eta_p^2 = .54$. For arithmetical facts we identified a significant main effect for time, $F(1,17) = 8.82, p = .009, \eta_p^2 = .34$, and for interaction, $F(1,17) = 4.42, p = .05, \eta_p^2 = .21$. Results were similar for numerical knowledge, with a main effect for time, $F(1,17) = 7.54, p = .01, \eta_p^2 = .31$, and for interaction, $F(1,17) = 4.31, p = .05, \eta_p^2 = .20$. In all cases, time effects revealed an improvement from pre to posttraining assessments, and significant interaction effects indicated a differential pre-post gain, favoring the experimental condition. In some cases, such as mental calculation time, students in the DYSC control group demonstrated lower scores (i.e., slower mental calculation) from pretest to the posttraining assessment.

Efficacy of the Treatment in Children With Mathematical Difficulties

We conducted the same statistical analysis for the MD group. Descriptive statistics are shown for the MD group in Table 3. A significant main effect of time, $F(1,33) = 9.21, \eta_p = .005, \eta_p^2 = .22$, was observed for accuracy in mental calculation; no statistically significant effect was observed for
time in mental calculation. For written calculation, we identified a significant main effect for time, $F(1,33) = 79.18, p < .001, \eta^2_p = .71$, and time x condition interaction, $F(1,33) = 41.96, p < .001, \eta^2_p = .56$. For arithmetical facts, we again identified a significant main effect for time, $F(1,33) = 23.6, p < .001, \eta^2_p = .42$, and an interaction effect, $F(1,33) = 17.92, p < .001, \eta^2_p = .35$. For numerical knowledge, analysis revealed a main effect for time, $F(1,33) = 22.03, p < .001, \eta^2_p = .4$, and time by condition interaction, $F(1,33) = 16.07, p < .001, \eta^2_p = .33$.

Again, all observed significant pre-post differences favored the post-tests, and significant interaction effects indicated that students in the experimental individualized training condition improved their performances to a greater extent than students in the control condition.

**Clinical Change**

To evaluate the validity of the training, we identified several criteria for quantitatively evaluating clinical improvement seen in several mathematical areas trained, based upon the guidelines produced by the Consensus Conference on Learning Disabilities (2010). An improvement of at least 1 standard deviation was defined as a significant clinical change. The frequencies of the students whose performance improved were calculated from a $z$ score of $-2$ to a $z$ score of $-1$ (or less). Transition from more problematic to less problematic levels suggests a clinically significant improvement.

This type of analysis was employed because a change that is not statistically significant for the group is sometimes important for the individual in clinical terms; this type of analysis reveals even minimal improvements that may be negligible when group averages are analyzed, but may be very important for the individual student.

Table 4 displays the effect sizes of the comparisons between the percentages of participants meeting the clinical criteria for a positive change, by type of training (experimental vs. control) and group (DYSC vs. MD). The number of participants for each task corresponds to the number of participants whose performance was two standard deviations below average at the initial assessment. Based on the clinical significance criteria, individualized training clearly improved students’ performance in both the DYSC and the MD groups, compared with controls in all parameters except for mental calculation time, with an effect size (ES) ranging from 0.32 (for mental calculation errors) to 1.56 (for arithmetical facts), mean ES = 0.99, SD = 0.49.

Students who benefited most were those with less severe mathematical disabilities who received the experimental individualized training (MD experimental group), followed by students with DYSC who received the individualized training (DYSC experimental group), whereas few of the
control students (DYSC control group and MD control group) experienced any specific improvement (i.e., $z$ scores changing from −2 to −1 or less).

**Follow-Up Analysis**

A sample of the students who received the experimental individualized training received a follow-up assessment 4 months after completing the training, including six students with DYSC and eight students with MD. The remaining students were unavailable for various reasons (e.g., change of school, change of residence). Unfortunately, it was not possible to involve the schools in the follow-up assessment. Table 5 illustrates the results of the follow-up assessment in terms of clinical improvement, that is, calculating the frequencies of students whose $z$ scores improved from −2 to −1 or less between the pre and posttraining and the follow-up assessment.

Data presented in Table 5 indicate a marked improvement in mental calculation errors and good stability of these results for both groups.

**Table 3**

<table>
<thead>
<tr>
<th>Variable</th>
<th>MD Experimental Group (N = 17)</th>
<th>MD Control Group (N = 18)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
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<tr>
<td>Mental calculation errors</td>
<td>Pre</td>
<td>3.41</td>
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<tr>
<td></td>
<td>Post</td>
<td>2.18</td>
</tr>
<tr>
<td>Mental calculation errors</td>
<td>Pre</td>
<td>76.35</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>71.41</td>
</tr>
<tr>
<td>Written calculation</td>
<td>Pre</td>
<td>2.59</td>
</tr>
<tr>
<td>(correct answers)</td>
<td>Post</td>
<td>5.41</td>
</tr>
<tr>
<td>Arithmetical facts errors</td>
<td>Pre</td>
<td>6.47</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>3.24</td>
</tr>
<tr>
<td>Numerical knowledge</td>
<td>Pre</td>
<td>15.53</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>19.06</td>
</tr>
</tbody>
</table>

MD = mathematical difficulties

**Table 4**

<table>
<thead>
<tr>
<th>Task</th>
<th>Group</th>
<th>Experimental</th>
<th>Control</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mental calculation errors</td>
<td>Dyscalculia</td>
<td>3/10 (30%)</td>
<td>1/5 (20%)</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td>8/12 (66.66%)</td>
<td>3/10 (30%)</td>
<td>0.95</td>
</tr>
<tr>
<td>Mental calculation time</td>
<td>Dyscalculia</td>
<td>1/9 (11.11%)</td>
<td>0/9 (0%)</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td>4/12 (33.33%)</td>
<td>2/10 (20%)</td>
<td>0.41</td>
</tr>
<tr>
<td>Written calculation</td>
<td>Dyscalculia</td>
<td>6/8 (75%)</td>
<td>2/7 (28.57%)</td>
<td>1.24</td>
</tr>
<tr>
<td>(correct answers)</td>
<td>MD</td>
<td>13/16 (81%)</td>
<td>3/9 (33.33%)</td>
<td>1.32</td>
</tr>
<tr>
<td>Arithmetical facts errors</td>
<td>Dyscalculia</td>
<td>2/10 (20%)</td>
<td>0/7 (0%)</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td>13/16 (81%)</td>
<td>3/12 (25%)</td>
<td>1.56</td>
</tr>
<tr>
<td>Numerical knowledge</td>
<td>Dyscalculia</td>
<td>3/5 (60%)</td>
<td>1/6 (16.66%)</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td>5/7 (71.43%)</td>
<td>1/6 (16.66%)</td>
<td>1.53</td>
</tr>
</tbody>
</table>

DYSE = dyscalculia    MD = mathematical difficulties
Half of the DYSC and MD students changed their performance from –2 to –1 z score and maintained this improvement over time. The same pattern was seen in the results for calculation time in the DYSC students, although the MD group improved already at the posttraining assessment with further improvement at the follow-up assessment (only three students’ z score remained below 2). For written calculation, the experimental MD group preserved the improvement seen after the training (all these students’ performance improved and this result was maintained), however, in the DYSC experimental group, the performance at the final follow-up deteriorated in one case. For the arithmetical facts, the pattern of the results differed between the two groups after the individualized training: all the MD students’ performance improved and this result was maintained at the follow-up, but the DYSC students’ improvement after the training was no longer apparent at the subsequent follow-up. For the numerical knowledge parameter, both trained groups improved and this result was maintained. However, one student in each group scored lower on the later follow-up.

**DISCUSSION**

The present investigation was intended to determine the nature and focus of training in mathematical skills for primary school students with either DYSC or mild MD, not meeting the criteria for a diagnosis of DYSC. This study compared two types of mathematical training: one specific (experimental individualized training) and the other more general (control training). The former was based upon a detailed analysis of each child’s mathematical learning profile, followed by specific individualized training intended to improve students’ more severely impaired mathematical skills. The latter, general training, involved specific sessions of relevant after-school mathematics activities. Half of the students in each group (DYSC and MD) received specific individualized training while the others received general training; this enabled us to ascertain the efficacy of the specific training in students with different levels of math-related learning difficulties.

Our results showed that the individualized training was beneficial in both groups: the DYSC students improved significantly after the training in mental calculation, and the MD students improved, especially in written calculation. When the students’ performance before and after the training was analyzed, only the students given the individualized training had improved (according to the criteria expressed in the Consensus Conference on Learning Disabilities guidelines, 2010); the controls who followed the general training did not (with respect to time taken in the mental cal-

---

### TABLE 5

<table>
<thead>
<tr>
<th>Task</th>
<th>Group</th>
<th>Pre –2 z</th>
<th>Pre –1 z</th>
<th>Post –2 z</th>
<th>Post –1 z</th>
<th>Follow-Up –2 z</th>
<th>Follow-Up –1 z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mental calculation errors</td>
<td>Dyscalculia</td>
<td>6</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Mental calculation time</td>
<td>Dyscalculia</td>
<td>5</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Written calculation</td>
<td>Dyscalculia</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>(correct answers)</td>
<td>MD</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Arithmetical facts errors</td>
<td>Dyscalculia</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>MD</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Numerical knowledge</td>
<td>Dyscalculia</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>5</td>
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<tr>
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<td>MD</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>8</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

DYSE = dyscalculia     MD = mathematical difficulties

(DYSC experimental group and MD experimental group). Half of the DYSC and MD students changed their performance from –2 to –1 z score and maintained this improvement over time.

The same pattern was seen in the results for calculation time in the DYSC students, although the MD group improved already at the posttraining assessment with further improvement at the follow-up assessment (only three students’ z score remained below 2). For written calculation, the experimental MD group preserved the improvement seen after the training (all these students’ performance improved and this result was maintained), however, in the DYSC experimental group, the performance at the final follow-up deteriorated in one case. For the arithmetical facts, the pattern of the results differed between the two groups after the individualized training: all the MD students’ performance improved and this result was maintained at the follow-up, but the DYSC students’ improvement after the training was no longer apparent at the subsequent follow-up. For the numerical knowledge parameter, both trained groups improved and this result was maintained. However, one student in each group scored lower on the later follow-up.
When the students’ performance before and after the training was analyzed, only the students given the individualized training had improved; the controls who followed the general training did not (with respect to time taken in the mental calculation task, their performance became even worse).

Our data suggest that general training in mathematics is of limited utility; conversely, training tailored to the child’s mathematical disability profile can produce positive results. By comparison with other similar studies that focused on specific mathematical areas, such as counting (Fuchs et al., 2010) or mental number line (Kucian et al., 2011), our study assumed a more general perspective with an ecological impact. We considered mathematical areas of significant importance across several grade levels: numerical knowledge, automaticity in retrieving and using arithmetical facts, mental calculation, and written calculation. In our experimental training, we tailored our intervention to each child’s individual mathematical profile, rather than adopting a specific mathematical instructional procedure, as done in the majority of the published studies (Gersten et al., 2009). As Dowker emphasized (2005), an accurate initial assessment is important to identify the strengths, weaknesses, and educational needs of individuals or groups, and to assess the effectiveness of any intervention.

The results of our follow-up assessment on a sample of the students given individualized training showed that the positive results, seen after the training, were durable in most cases, although gains realized by students with DYSC tended to deteriorate somewhat over time.

The present study has some limitations, including the relatively small number of participants involved. To be effective, our training could only be provided for small groups of students, making it more difficult to generalize our findings. In the future, we hope to collect more data to enlarge our sample and make our findings more generalizable. Another important issue that might be addressed in future studies concerns the heterogeneity of the children’s clinical and chronological characteristics. Finally, because of the way our center is organized, psychologists worked with the experimental group and educators worked with the control group, and this may have influenced outcomes to some extent. To minimize this risk, we selected educators with a great deal of experience working with children with LD, who had attended a masters’ course on LD psychopathology and/or worked with children with LD for many years. In addition, these educators and the psychologists involved in the study were all constantly supervised by the psychologist author (Dr. Martina Pedron) of the present article.

Even considering these limitations, our findings lead us to conclude that specific training adapted to each child’s cognitive profile is a better solution for effective training purposes, and that the results obtained are generally durable, even in students with DYSC. The greatest improvements (sometimes at or approaching average achievement levels) were typically observed in students with milder math learning difficulties.

Overall, our investigation provides implications in clinical, educational, and theoretical contexts. Clinically, our results suggest that resistance to intervention may be an important indicator of DYSC, and this fact may allow for more precise diagnosis of this condition, avoiding false positives (as many as 17% of the student population, according to Lucangieli & Cornoldi, 2007). Specifically, early training is effective in such cases because, in addition to solving any diagnostic concerns, it prevents children from falling behind in school and promotes more positive outcomes, leading to greater levels of success in mathematics.
In addition, our findings have educational implications, in that specific training, based upon a given student’s mathematical learning profile can be genuinely effective. On the other hand, more generic, curriculum-based support based generally upon additional time and practice may be ineffective. Finally, our findings provide theoretical implications in that they provide evidence of the reciprocal interaction between education science and cognitive science. That is, with an appropriate educational system, it is possible to modify students’ individual potentialities. Further research could provide further evidence of the benefits of specialized, individualized training. At present, our results suggest that this training can be an effective means of addressing different levels of mathematics difficulties.

**REFERENCES**


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Manuscript received August 2012; accepted January 2013.